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OPTIMAL PRICING WITH PRODUCT QUALITY DIFFERENCES AND CONTRACTUAL PENALTIES

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Abstract

There is quite a debate between legal scholars from Civil and Common Law systems on whether contractual penalties should be enforced by courts. In this contribution we look at this question from a different angle by modelling a monopolist trying to find optimal prices when contracts without penalty as well as contracts with penalties are offered simultaneously to a class of buyers who face different levels of loss risks. We find that neither changes in legal indemnities nor in privately stipulated fines alter those prices. Also, seller's profit and consumer surplus are not influenced by such changes. Therefore we conclude that full enforcement of penalty clauses as agreed upon is the best recommendation in order to reap benefits like optimal inter-party risk-allocation.

JEL classification: K12.

1 Introduction

Most legal systems offer the possibility of stipulating liquidated damages and/or contractual penalties. Such clauses state in advance what amount of money has to be paid if one party fails to perform as agreed upon. These provisions allow to

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amend the legal rules and allocate the risk according to the parties' needs tailoring the incentive structure for a specific contract (Triantis 2000, Cooter/Ulen 2000, p. 235-237, Polinsky 1989, p. 63-65). Regarding such clauses there is a remarkable difference between Civil Law and Common Law tradition. In Common Law contractual penalty clauses (exceeding the actual damage) are not enforceable. Many Law & Economics scholars criticize Common Law courts for the non-enforcement but a minority of scholars defend the attitude of Common Law towards penalty clauses (Hatzis 2003). Interestingly also courts in the Civil Law tradition have adopted a skeptical attitude towards stipulated fines so that the differences between the two legal traditions is in fact small.

Common Law may explicitly prohibit enforcement of contractual penalties in general but it allows for liquidated damages clauses. In contrast, Civil Law permits penalty clauses in principle but allows judges to reduce the stipulated amount of fine ex post. Therefore, in fact, even in Civil Law we usually find that pure penalties (exceeding a reasonable estimate of the damage) won't be executed by courts.[1]

This is a quite astonishing position. Absent some reason to believe that one or both parties misunderstood the contract or was (either physically or economically) forced to accept the clause, one would normally assume that they incurred the costs of specifying this clause because it was Pareto superior to the outcome which could have been achieved through law itself (Mahoney 2000, Cooter/Ulen 2000, p. 236). In particular it is not true that the mere existence of a penalty clause is an indicator of fraud or mistake. There are plausible conditions under which both parties are better off under the regime of a penalty clause.

The economics of contractual penalties have so far been studied with respect to optimal risk allocation and contract breach decisions (e.g. Spier/Whinston 1995, Triantis/Triantis 1998, Che/Chung 1999). In general, penalty clauses offer benefits for both parties. On the one hand, a pending fine acts as an additional incentive to perform correctly. This reflects the risk allocation property of a penalty agreement. On the other hand, the obligor can signal his reliability (DeGeest/Wuyts 2000).

Despite contributions regarding such topics, aspects of how allowing for dif-

[1]In Austria and Germany for example the use of penalty clauses is allowed but both national laws concede that courts may abate the stipulated fine (cp. § 1336 Austrian General Civil Rights Code and §§ 339-343 German Civil Rights Code).

ferent levels of penalties affects business decisions have been quite overlooked in the literature so far. Given that in some branches of industry penalty clauses are abundant this absence of a body of research is striking.

Therefore the aim of this paper is twofold: On the one hand we want to show that allowing for contracts to include penalty clauses doesn't lead to welfare losses. Therefore we find that penalty agreements should be enforced as contracted. On the other hand we derive the optimal prices for a monopolistic seller offering a mix of two types of contracts. This contributes to the neglected discussion on business effects of stipulated fines.

2 Model setup

Before going into detail we want to provide an overview of the model:

We consider a monopolist selling a good to a group of buyers. The good is homogeneous except for the probability of being defective. If the seller invests more into diligence he can lower the probability of failure. Buyers differ in the impact of defects of the good.

The seller offers two types of contracts: one with a lower price for the good with higher probability of failure and one with a higher price for the product with lower probability of being defective. Furthermore the high-price contract includes a penalty clause whereas the low-price buyers only get indemnities as granted by court. Prices are chosen by the seller. Buyers can only choose which contract to conclude or not to buy at all. The amount of penalty is an exogenous parameter.

We will now introduce the parameters and variables of the model in detail:

A monopolistic seller can choose between two different levels of care such that different probabilities for the good being defective result. We denote π_H the probability of failure with a high level of care and π_L the probability with low diligence with $0 < \pi_H < \pi_L < 1$. Choosing a higher level of care leads to higher production costs; the seller faces variable production costs in the amount of $c_L > 0$ for low diligence and $c_H > c_L$ per item for high level of care. The different probabilities are common knowledge whereas the costs are the monopolist's private information.

The seller is confronted with a group of potential buyers who are characterized by different levels of losses λ in case of defections of the good whereas their benefit $\beta > c_H$ from using a faultless good is assumed to be equal for all buyers and high enough such that even high-quality production is socially optimal. The number of buyers is normalized to unit and the potential loss from having to use a defective good is uniformly distributed on the interval $[0, \bar{\lambda}]$. The good may still operate if failures arise but not properly and buyers are affected differently by the limited usability of a defective good. The benefit is common knowledge whereas the potential loss is the buyer's private information. Any defects of the good become evident as soon as the buyer starts using it. So we don't have to consider effects of time or usage intensity.

Now the seller wants to design two types of contracts in order to maximize his overall (expected) profit. The monopolist can either charge $p_0 > 0$ for the good with no additional penalty clause or he demands $p_F > p_0$ in exchange for an agreement to pay a stipulated fine F if the good is defective. We assume perfect correlation, i.e. the seller uses high diligence for contracts with stipulated fines and low care for those without such an agreement.

If a good is defective the buyer gets compensated either by an indemnity payment I granted by court (and therefore exogenously given) if no fine has been stipulated or by the contractual penalty F as agreed upon. The amount of that fine is fixed and given exogenously. This assumption can be explained thinking about the courts' skeptical attitude to enforce privately stipulated penalties. Therefore only F is known to be executable. So buyers who want a penalty ask for the maximum amount possible which is F but they cannot influence this parameter. Since some types of buyers will only buy if they are additionally insured through such a penalty clause the seller has an incentive to offer an alternative contract for those types.

All the buyers have the same utility function (being linear in monetary terms which implies risk-neutrality).

For a good bought without penalty agreement:

$$U_0 = \beta - p_0 - \pi_L(\lambda - I) \tag{1}$$

For a contract including a stipulated fine:

$$U_F = \beta - p_F - \pi_H(\lambda - F) \tag{2}$$

These utility functions help us to derive conditions for which buyers will choose what kind of contracts. From $U_F > U_0$ we find the type of the critical buyer who is just indifferent between choosing either kind of contract and from $U_0 > 0$ and $U_F > 0$ we can find the threshold types of buyers who will be just indifferent between buying and abstaining.

$$U_0 > 0 \Rightarrow \lambda_0 = \frac{\pi_L I - p_0 + \beta}{\pi_L} \tag{3}$$

$$U_F > 0 \Rightarrow \lambda_F = \frac{\pi_H F - p_F + \beta}{\pi_H} \tag{4}$$

$$U_F > U_0 \Rightarrow \hat{\lambda} = \frac{p_F - p_0 + \pi_L I - \pi_H F}{\pi_L - \pi_H} \tag{5}$$

We assume that $0 \leq \hat{\lambda} < \lambda_0 < \lambda_F \leq \bar{\lambda}$ which leads to the most intuitive distribution of contract. Then buyers facing a potential loss $\lambda \in (\lambda_F, \bar{\lambda}]$ will not buy the good at all as their expected utility is negative for both types of contract because of the high loss. Buyers with $\lambda \in [\hat{\lambda}, \lambda_F]$ will opt for the contract including a penalty clause. In that group we have buyers who would take the good only if the contract provided for a stipulated fine and those who would also take the good without such a clause but find it more profitable to opt for its inclusion into the contract. The last group consists of those buyers facing very low potential losses $\lambda \in [0, \hat{\lambda})$ who will take the low-price conract without penalty clause anyway. (Appendix A.1 summarizes the conditions for the parameters to fulfill this assumption.)

$$\begin{aligned} \max_{p_0, p_F} V = \quad & \tfrac{\hat{\lambda}}{\bar{\lambda}}(p_0 - c_L - \pi_L I) + \tfrac{\lambda_F - \hat{\lambda}}{\bar{\lambda}}(p_F - c_H - \pi_H F) \\ & \text{s.t.} \\ & c_L - p_0 \leq 0 \\ & c_H - p_F \leq 0 \\ & -\tfrac{\hat{\lambda}}{\bar{\lambda}} \leq 0 \\ & \tfrac{\hat{\lambda} - \lambda_F}{\bar{\lambda}} \leq 0 \\ & p_0 \geq 0 \\ & p_F \geq 0 \end{aligned} \tag{6}$$

3 Optimal prices and contract distribution

Substituting the threshold values and including the Kuhn-Tucker-multipliers we rewrite the profit function as follows:

$$
\begin{aligned}
\tilde{V} = \frac{p_F - p_0 + \pi_L I - \pi_H F}{(\pi_L - \pi_H)\bar{\lambda}}(p_0 - c_L - \pi_L I) + \\
\frac{(\pi_L - \pi_H)(\pi_H F - p_F + \beta) - \pi_H(p_F - p_0 + \pi_L I - \pi_H F)}{\pi_H(\pi_L - \pi_H)\bar{\lambda}} \times \\
(p_F - c_H - \pi_H F) + \eta_1(p_0 - c_L) + \eta_2(p_F - c_H) + \\
\eta_3(\frac{p_F - p_0 + \pi_L I - \pi_H F}{(\pi_L - \pi_H)\bar{\lambda}}) + \\
\eta_4(\frac{(\pi_L - \pi_H)(\pi_H F - p_F + \beta) - \pi_H(p_F - p_0 + \pi_L I - \pi_H F)}{\pi_H(\pi_L - \pi_H)\bar{\lambda}})
\end{aligned}
\tag{7}
$$

>From this function we find the Kuhn-Tucker conditions and have to solve the following system of equations in order to get the optimal prices. (In appendix B.1 we test for sufficiency of the Kuhn-Tucker conditions.)

$$
\begin{aligned}
\frac{\partial \tilde{V}}{\partial p_0} = -2p_0 + 2p_F - 2\pi_H F + 2\pi_L I - c_H + c_L + \\
\eta_1(\pi_L - \pi_H)\bar{\lambda} - \eta_3 + \eta_4 \leq 0 \\
\frac{\partial \tilde{V}}{\partial p_F} = \pi_H(2p_0 - c_L - 2\pi_L I) - \pi_L(2p_F - c_H - 2\pi_H F) + \\
(\pi_L - \pi_H)(\beta + \eta_2 \pi_H \bar{\lambda}) + \pi_H \eta_3 - \pi_L \eta_4 \leq 0 \\
p_0 \geq 0 \quad p_F \geq 0 \quad p_0 \tfrac{\partial \tilde{V}}{\partial p_0} = 0 \quad p_F \tfrac{\partial \tilde{V}}{\partial p_F} = 0 \\
\tfrac{\partial \tilde{V}}{\partial \eta_1} = p_0 - c_L \geq 0 \qquad \eta_1 \geq 0 \quad \eta_1 \tfrac{\partial \tilde{V}}{\partial \eta_1} = 0 \\
\tfrac{\partial \tilde{V}}{\partial \eta_2} = p_F - c_H \geq 0 \qquad \eta_2 \geq 0 \quad \eta_2 \tfrac{\partial \tilde{V}}{\partial \eta_2} = 0 \\
\tfrac{\partial \tilde{V}}{\partial \eta_3} = p_F - p_0 + \pi_L I - \pi_H F \geq 0 \quad \eta_3 \geq 0 \quad \eta_3 \tfrac{\partial \tilde{V}}{\partial \eta_3} = 0 \\
\frac{\partial \tilde{V}}{\partial \eta_4} = \pi_L(\pi_H F - p_F) - \pi_H(\pi_L I - p_0) + (\pi_L - \pi_H)\beta \geq 0 \\
\eta_4 \geq 0 \quad \eta_4 \tfrac{\partial \tilde{V}}{\partial \eta_4} = 0
\end{aligned}
\tag{8}
$$

Without loss of generality we can set $\eta_1 = \eta_2 = \eta_3 = \eta_4 = 0$. This also makes sense since otherwise our seller wouldn't have any room to set prices. Furthermore we know that $P_F > P_0 > 0$. In order to fulfill all constraints (and find an interior solution to this maximization problem) we have to solve the following system of equations:

$$\begin{aligned} -2p_0 + 2p_F &= 2\pi_H F - 2\pi_L I + c_H - c_L \\ 2\pi_H p_0 - 2\pi_L p_F &= \pi_H(c_L + 2\pi_L I) - \pi_L(c_H + 2\pi_H F) - (\pi_L - \pi_H)\beta \end{aligned} \tag{9}$$

This system yields a unique non-trivial solution for p_0 and p_F as the determinant $\begin{bmatrix} -2 & 2 \\ 2\pi_H & -2\pi_L \end{bmatrix} = 4(\pi_L - \pi_H)$ is positive and the equations are independent. Applying Cramer's rule we get the following optimal prices.

$$p_0^* = \frac{c_L + \beta + 2\pi_L I}{2} \tag{10}$$

$$p_F^* = \frac{c_H + \beta + 2\pi_H F}{2} \tag{11}$$

Certainly we have $p_F^* > p_0^* > 0$ because of $c_H > c_L$ and $F > I > 0$. But for this solution to be interior we have to demand that the parameters meet some additional criteria. (In appendix A.2 we show that these optimal prices are consistent with the assumptions for the parameters under some auxiliary conditions such that the threshold values for the buyers have the correct ordering.)

Using these optimal prices we can now find the contracts' distribution among the buyers. Therefore we define as λ_L^* the percentage of consumers buying low-price variety and we denote λ_H^* the proportion of buyers opting for the high-price contract including a penalty clause.

$$\lambda_L^* = \frac{\hat{\lambda}}{\bar{\lambda}} = \frac{p_F^* - p_0^* + \pi_L I - \pi_H F}{\bar{\lambda}(\pi_L - \pi_H)} = \frac{c_H - c_L}{2(\pi_L - \pi_H)\bar{\lambda}} \tag{12}$$

$$\lambda_H^* = \frac{\lambda_F - \hat{\lambda}}{\bar{\lambda}} = \frac{\beta(\pi_L - \pi_H) - \pi_L p_F^* + \pi_H p_0^* + \pi_L \pi_H (F - I)}{\pi_H(\pi_L - \pi_H)\bar{\lambda}} = \frac{\beta}{2\pi_H \bar{\lambda}} + \frac{\pi_H c_L - \pi_L c_H}{2\pi_H(\pi_L - \pi_H)\bar{\lambda}} \tag{13}$$

Throughout the following discussion and analysis we assume that the parameters meet the qualifications summarized in table 1. (Appendix A.3 provides the remaining derivations for those conditions.) Under those assumptions both types of contracts are offered by the seller and exhibit positive demand. We can confine our further analysis to a situation where those conditions hold. Other constellations would yield results for cases where only one of the contract types is offered leading to simple profit maximizations for the monopolist. But we want to study how a seller offering both kinds of contracts reacts on changes in the parameters (as long as they stay in the assumed domain). In the next section we provide a general discussion of our results and a numeric example is presented in appendix C.

Condition	**Purpose**
Assumed conditions	
$0 < \pi_H < \pi_L < 1$	Higher product quality is defined by lower probability of break-down.
$c_H > c_L$	Higher-quality goods are more costly to produce.
$p_F > p_0 > 0$	Contracts with penalty clauses are more expensive for the buyer. (Perfect correlation between price and quality is assumed.)
$\beta > c_H$	Buyers' benefit (exogenously given) is positive and high-quality production is socially optimal.
$\lambda \in [0, \bar{\lambda}]$	Potential loss is uniformly distributed on the given interval with $\bar{\lambda} > 0$
$F > I > 0$	Privately stipulated penalties are higher than indemnity payments granted by courts.
Derived conditions for an interior optimum at p_0^*, p_F^*	
$2\bar{\lambda} \geq \frac{\beta - c_H}{\pi_H} > \frac{\beta - c_L}{\pi_L}$	Both contracts are offered by the seller; threshold ordering is as assumed; both groups of buyers get non-negative utility.
$\beta \leq c_H + 2\pi_H\bar{\lambda}$	Total demand cannot exceed 100 percent of buyers.

Table 1: Parameter conditions

4 Discussion

Obviously both optimal prices increase in benefit and costs. This is clear since the monopolist tries to reap the consumer surplus. So a higher benefit will lead the seller to demand higher prices. Also both compensatory payments lead to a price increase in the amount of the expected compensation per contract. Therefore we see either the indemnity's or the penalty's full expectation value in the optimal prices. Demanding higher penalties leads to higher prices for contract including such a fine. But so does increasing indemnities. If courts become more rigorous with liability claims and grant higher indemnities, then prices will rise too. So there is no difference between those two remedies against product failures.

The percentages of consumers buying either kind of contract depend on several determinants. On the one hand there are both the production cost difference and the difference between the probabilities failure which come into play because the seller decides on the prices partly on behalf of those factors and that in turn influences the number of buyers for each contract type. On the other hand there is the potential loss' upper limit in both percentages and the benefit in the proportion in the high-price contract consumers. An increase of the boundary value for the potential loss leads to lower numbers of buyers in both categories which is clear since ceteris paribus higher potential losses lead to less expected utility for both groupy of consumers. Higher benefits on the contrary lead only to more buyers joining the high-price group without diminishing the low-price group. (Appendix D.1 provides for the necessary comparative statics.)

Another interesting point is that neither the seller's profit nor the consumers' surplus changes if the amount of indemnity or penalty is changed. This is clear because using the optimal prices cancels out the expectation values of those compensatory payments and the optimal buyers' distribution among contract types is not dependent on those payments. (Additionally appendices D.2 and D.3 demonstrate in more detail why F and I are irrelevant.) Thinking about the fact that many companies claim that higher penalty diminish their profits and should therefore be abolished, this becomes a quite remarkable result. It shows that such lobbyism has no solid ground as long as companies find a positive demand for contracts without penalties too. Then those firms who cannot (or don't want to) afford penalty agreements can still serve the low-price segment among buyers. It also shows that allowing for higher penalties to be enforced by courts doesn't change the overall consumer surplus. So there is no danger that higher penalties make sellers or buyers worse off.

At last from a comparison of this situation where both types of contracts are allowed and the outcome where only one of those types is admitted we can see that the prices don't change but that less consumers are served. (This is derived in appendix B.2 and the numeric example in appendix C includes it as well.) So forbidding privately stipulated penalties doesn't make goods cheaper but leads to less consumers buying the product. On the other hand solely relying on penalty agreements cannot be suggested either since then all buyers would have to pay the same high price. The best recommendation then is to provide a certain "base protection" for buyers through indemnities granted by court and let those buyers who need better insurance privately stipulate fines with their contract partners. Then, of course, such clauses have to be executed by courts.

5 Summary and further research

We started out to show that penalty clauses are not detrimental to social welfare and to look at the business implications if courts enforce higher amounts of privately stipulated penalties.

>From the model's results we have seen that allowing the seller to offer both kinds of contracts leads to different prices with the price for a contract including a penalty clause being higher. But since only those buyers opt for the high-price contract who would suffer from a high loss in case of product failure this only underlines the positive effects of such clauses to provide for an optimal inter-party risk allocation based on the idiosyncratic values at stake. Prohibiting penalty clauses would lead to less overall demand. So some buyers who would gain positive utility from concluding the high-price contract leave the market unserved because of the lack of insurance against their potential loss. This backs up our view that penalty clauses exhibit positive effects for both parties to a contract and should be enforced by courts.

Regarding business implications we found that despite prices changes due to shifts in the parameters "indemnity" and "fine", seller's profit is not altered since the demand adjusts according to the altered prices. This contradicts popular claims on behalf of companies that allowing for higher penalties to be executed by courts would ruin firms.[2]

Summing all up, we derived a very positive result in favor of full enforcement of contractual penalty clauses. Of course, we simplified our model in some ways. Although we were still able to derive some qualitative findings there is ample room for further research. One could alleviate the assumption of perfect correlation between price and quality including potential moral hazard on the seller's side. Also the monopoly assumption could be altered into an oligopoly or perfect competition setting allowing for strategic interaction. At last, the amount of penalty could be endogenized such that the buyer sets the penalty and the seller can either accept at a certain price or exit. This would result in a principal-agent-situation.

Appendix A: Conditions for the parameters

Appendix A.1: Critical thresholds' ordering

We assumed in the text that $0 \leq \hat{\lambda} < \lambda_0 < \lambda_F \leq \bar{\lambda}$. Using the definitions of those threshold values we can rewrite that condition as follows.

$$0 \leq \frac{p_F - p_0 + \pi_L I - \pi_H F}{\pi_L - \pi_H} < \frac{\pi_L I - p_0 + \beta}{\pi_L} < \frac{\pi_H F - p_F + \beta}{\pi_H} \leq \bar{\lambda} \qquad (14)$$

Both inner inequalities hold if the next condition is met which can be found after some trivial algebraic manipulations.

$$\frac{p_F}{\pi_H} - \frac{p_0}{\pi_L} < F - I + \frac{\pi_L - \pi_H}{\pi_H \pi_L}\beta \qquad (15)$$

The right-hand side of that expression is always positive since $F > I$, $\pi_L > \pi_H > 0$ and $\beta > 0$ by assumption. But also the left-hand side of the inequality is positive because of $p_F > p_0$ and $\pi_L > \pi_H > 0$. Therefore it is possible that parameter constellations exist which violate this condition and lead to abnormal ordering of the thresholds.

[2] In January 2003, for example, the German construction industry acclaimed a ruling of the German Federal Supreme Court limiting contractual penalties in general terms of trade to not more than 5% of the order value if no higher real damages could be proved.

The boundaries lead to very intuitive conditions for the price difference and the price for a contract with a penalty clause. The first states that the price difference has to be greater than the difference of the expected compensation payments. If this condition would not hold, then every buyer would opt for a penalty clause to be included in the contract as its price increase would be surpassed by the additional expected compensational payment. The second condition shows that the price for a contract with stipulated fine has to be higher than the benefit lesser the expected uncovered damages. If that would not be the case, then there would be no buyer abstaining from transaction.

$$p_F - p_0 \geq \pi_H F - \pi_L I \tag{16}$$

$$p_F \geq \beta - \pi_H(\bar{\lambda} - F) \tag{17}$$

At last we can easily check that at least the boundary prices $\tilde{p}_F = \beta - \pi_H(\bar{\lambda} - F)$ and $\tilde{p}_0 = \beta - \pi_H\bar{\lambda} + \pi_L I$ fulfill the above condition according to our assumptions.

$$\frac{\tilde{p}_F}{\pi_H} - \frac{\tilde{p}_0}{\pi_L} < F - I + \frac{\pi_L - \pi_H}{\pi_H \pi_L}\beta \tag{18}$$

$$\frac{\beta - \pi_H(\bar{\lambda} - F)}{\pi_H} - \frac{\beta - \pi_H\bar{\lambda} + \pi_L I}{\pi_L} < F - I + \frac{\pi_L - \pi_H}{\pi_H \pi_L}\beta \tag{19}$$

$$\frac{(\pi_L - \pi_H)\beta - \pi_H\pi_L(\bar{\lambda} - F - I) - \pi_H^2\bar{\lambda}}{\pi_H\pi_L} < \frac{\pi_H\pi_L(F - I) + (\pi_L - \pi_H)\beta}{\pi_H\pi_L} \tag{20}$$

$$-\bar{\lambda}\pi_H(\pi_L - \pi_H) < 0 \tag{21}$$

Appendix A.2: Correct threshold ordering with optimal prices

First we check whether the optimal prices meet our criteria for the right ordering of the critical buyer types' thresholds. For $p_0^* = \frac{c_L + \beta + 2\pi_L I}{2}$ and $p_F^* = \frac{c_H + \beta + 2\pi_H F}{2}$ we find that the first boundary conditions for the correct ordering is fulfilled because of our assumption that $c_H > c_L$.

$$p_F^* - p_0^* \;=\; \frac{c_L + \beta + 2\pi_L I}{2} - \frac{c_H + \beta + 2\pi_H F}{2} = \tag{22}$$

$$\frac{c_H - c_L + 2\pi_H F - 2\pi_L I}{2} =$$
$$\frac{c_H - c_L}{2} + \pi_H F - \pi_L I \geq \pi_H F - \pi_L I$$

For the second condition to hold we have to demand that $\bar{\lambda} \geq \frac{\beta - c_H}{2\pi_H}$. This can be seen from the following equation.

$$\begin{aligned} p_F^* &\geq \beta - \pi_H(\bar{\lambda} - F) \\ \frac{c_H + \beta + 2\pi_H F}{2} &\geq \beta - \pi_H(\bar{\lambda} - F) \\ \beta + c_H + 2\pi_H F &\geq 2\beta + 2\pi_H F - 2\pi_H \bar{\lambda} \\ -2\pi_H \bar{\lambda} &\leq -\beta + c_H \\ \bar{\lambda} &\geq \frac{\beta - c_H}{2\pi_H} \end{aligned} \tag{23}$$

At last we have to examine the condition for the thresholds' inner relationships.

$$\begin{aligned} \frac{p_F^*}{\pi_H} - \frac{p_0^*}{\pi_L} = \frac{c_H + \beta + 2\pi_H F}{2\pi_L} - \frac{c_L + \beta + 2\pi_L I}{2\pi_H} = \\ F - I + \beta\frac{\pi_L - \pi_H}{2\pi_L\pi_H} + \frac{\pi_L c_H - \pi_H c_L}{2\pi_H\pi_L} < F - I + \beta\frac{\pi_L - \pi_H}{\pi_L\pi_H} \Longleftrightarrow \\ \beta\frac{\pi_L - \pi_H}{2\pi_L\pi_H} > \frac{\pi_L c_H - \pi_H c_L}{2\pi_H\pi_L} \Leftrightarrow \pi_L(\beta - c_H) > \pi_H(\beta - c_L) \end{aligned} \tag{24}$$

This last condition can be rewritten as $\frac{\beta - c_H}{\pi_H} > \frac{\beta - c_L}{\pi_L}$. This expression can be economically interpreted too. The threshold values will only then have the assumed ordering if the weighted differences between costs and benefit are higher for contracts with stipulated fines. If this condition was violated there would be no buyers opting for the high-price contract.

Appendix A.3: Further conditions for an interior solution

In the main text and in the previous appendix we found that the optimal prices calculated above meet nearly all criteria under some added assumptions. There are four inequalities left from the Kuhn-Tucker conditions which we have to look at now.

$$p_0^* \geq c_L \Leftrightarrow c_L \leq \beta + 2\pi_L I \tag{25}$$

$$p_F^* \geq c_H \Leftrightarrow c_H \leq \beta + 2\pi_L F \tag{26}$$

$$\begin{aligned} p_F^* - p_0^* + \pi_L I - \pi_F = \\ \frac{c_H + \beta + 2\pi_H F}{2\pi_L} - \frac{c_L + \beta + 2\pi_L I}{2\pi_H} + \pi_L I - \pi_H F = \\ \frac{c_H - c_L}{2} > 0 \end{aligned} \tag{27}$$

$$\begin{aligned} \pi_L(\pi_H F - p_F^*) - \pi_H(\pi_L I - p_0^*) + (\pi_L - \pi_H)\beta = \\ \frac{1}{2} \cdot (2\pi_L\pi_H F - \pi_L\beta - \pi_L c_H - 2\pi_L\pi_H F - 2\pi_L\pi_H I + \pi_H\beta + \\ \pi_H c_L + 2\pi_L\pi_H I + 2\pi_L\beta - 2\pi_H\beta) = \\ \frac{\beta(\pi_L - \pi_H) - \pi_L c_H + \pi_H c_L}{2} > 0 \Longleftrightarrow \frac{\beta - c_H}{\pi_H} > \frac{\beta - c_L}{\pi_L} \end{aligned} \tag{28}$$

These conditions are fulfilled by the optimal prices without further premises and the last one leads to the same requirement as already stated in the appendix above. These requirements take care that both kinds of contracts yield positive profit. If those conditions would be violated the related contract wouldn't be chosen by the seller at all.

At last we have to ensure that buyers' utilities are positive at the optimal prices and that the sum of the fractions λ_H^* and λ_L^* is not greater than 1. For the first part we check $U_0 \geq 0$ for the low-price contract's marginal type $\hat{\lambda}$ because all buyers with losses below that point get higher utilities. This is repeated for $U_F \geq 0$ regarding the high-price contract's marginal type λ_F.

$$\begin{aligned} U_0 \geq 0 \Leftrightarrow \beta \geq c_L + 2\pi_L\lambda \\ \beta \geq c_L + 2\pi_L\hat{\lambda} \\ \beta \geq c_L + 2\pi_L\frac{p_F^* - p_0^* + \pi_L I - \pi_H F}{\pi_L - \pi_H} \\ \beta \geq c_L + 2\pi_L\frac{c_H - c_L}{2(\pi_L - \pi_H)} \end{aligned} \tag{29}$$

$$\begin{aligned}\beta(\pi_L - \pi_H) &\geq \pi_L c_H - \pi_H c_L \\ \frac{\beta - c_H}{\pi_H} &\geq \frac{\beta - c_L}{\pi_L}\end{aligned}$$

$$\begin{aligned}U_F \geq 0 \Rightarrow \beta &\geq c_H + 2\pi_L \lambda \\ \beta &\geq c_L + 2\pi_H \lambda_F \\ \beta \geq c_H + 2\pi_H &\frac{\pi_H F - p_F^* + \beta}{\pi_H} \\ \beta \geq c_H + 2\pi_H F - (c_H + \beta + 2\pi_H F) &+ 2\beta \\ \beta &\geq \beta\end{aligned} \tag{30}$$

We see that the first requirement results in the same condition as above and that the second one is always fulfilled with equality. Therefore no additional assumptions are necessary to ensure that both types of consumers gain non-negative utility from choosing the contract of the group they belong to.

Of course, it is not possible that the monopolist sells to more than 100 percent of the customers. Therefore we conclude by checking that $\lambda_L^* + \lambda_H^* \leq 1$.

$$\begin{aligned}\lambda_L^* + \lambda_H^* &\leq 1 \\ \frac{c_H - c_L}{2(\pi_L - \pi_H)\bar{\lambda}} + \frac{\beta}{2\pi_H \bar{\lambda}} + \frac{\pi_H c_L - \pi_L c_H}{2\pi_H(\pi_L - \pi_H)\bar{\lambda}} &\leq 1 \\ \pi_H(c_H - c_L) + \beta(\pi_L - \pi_H) + \pi_H c_L - \pi_L c_H &\leq 2\pi_H(\pi_L - \pi_H)\bar{\lambda} \\ \beta &\leq c_H + 2\pi_H \bar{\lambda}\end{aligned} \tag{31}$$

Appendix B: Optimality conditions

Appendix B.1: Sufficiency of the Kuhn-Tucker conditions

In order that the Kuhn-Tucker conditions mentioned in the text are sufficient for a maximum at the point p_0^*, p_F^* found from the equational system we have to check that the objective function V is concave and that the restriction functions are convex.

>From simple inspection we find that all restriction functions are (weakly) convex in the decision variables as all partial second derivatives vanish. Therefore we

only have to check the objective function for concavity. Using the definition of the profit function and the threshold values from the text we can calculate the following partial derivatives:

$$\frac{\partial V}{\partial p_0} = \tfrac{-p_0+c_L+\pi_L I}{(\pi_L-\pi_H)\bar{\lambda}} + \tfrac{p_F-p_0+\pi_L I-\pi_H F}{(\pi_L-\pi_H)\bar{\lambda}} + \tfrac{p_F-c_H-\pi_H F}{(\pi_L-\pi_H)\bar{\lambda}} = 0 \quad (32)$$

$$\frac{\partial V}{\partial p_F} = \tfrac{p_0-c_L-\pi_L I}{(\pi_L-\pi_H)\bar{\lambda}} - \tfrac{\pi_L(p_F-c_H-\pi_H F)}{\pi_H(\pi_L-\pi_H)\bar{\lambda}} + \quad (33)$$

$$\tfrac{(\pi_L-\pi_H)(\pi_H F-p_F+\beta)-\pi_H(p_F-p_0+\pi_L I-\pi_H F)}{\pi_H(\pi_L-\pi_H)\bar{\lambda}} = 0$$

$$|J| = \begin{vmatrix} \frac{\partial^2 V}{\partial p_0^2} & \frac{\partial^2 V}{\partial p_0 \partial p_F} \\ \frac{\partial^2 V}{\partial p_F \partial p_0} & \frac{\partial^2 V}{\partial p_F^2} \end{vmatrix} = \begin{vmatrix} -\frac{2}{(\pi_L-\pi_H)\bar{\lambda}} & \frac{2}{(\pi_L-\pi_H)\bar{\lambda}} \\ \frac{2}{(\pi_L-\pi_H)\bar{\lambda}} & -\frac{2\pi_L}{\pi_H(\pi_L-\pi_H)\bar{\lambda}} \end{vmatrix} \quad (34)$$

The first principal minor is negative because of $\pi_L > \pi_H$ and $\bar{\lambda} > 0$. The full determinant stated below is positive because of the same assumption. So the Jacobian is negative definite and the objective function is concave. Thus the Kuhn-Tucker conditions are sufficient for a maximum.

$$\frac{4\pi_L}{\pi_H(\pi_L-\pi_H)^2\bar{\lambda}^2} - \frac{4}{(\pi_L-\pi_H)^2\bar{\lambda}^2} = \frac{4(\pi_L-\pi_H)}{\pi_H(\pi_L-\pi_H)^2\bar{\lambda}^2} \quad (35)$$

Appendix B.2: Optimal prices with only one contract

To calculate the optimal price if only one contract type was allowed we use the following profit functions (V_0 for contracts without penalty and V_F for those with) keeping in mind that the proportion of buyers is now given by λ_0 and λ_F respectively.

$$\max_{p_0} V_0 = \frac{\lambda_0}{\bar{\lambda}}(p_0 - c_L - \pi_L I) \quad (36)$$

$$V_0' = -2p_0 + c_L + \beta + 2\pi_L I = 0 \Rightarrow p_0^x = \frac{c_L + \beta + 2\pi_L I}{2} = p_0^* \quad (37)$$

$$\max_{p_F} V_F = \frac{\lambda_F}{\bar{\lambda}}(p_F - c_H - \pi_L F) \quad (38)$$

$$V_F' = -2p_F + c_H + \beta + 2\pi_H F = 0 \Rightarrow p_F^x = \frac{c_H + \beta + 2\pi_H F}{2} = p_F^* \quad (39)$$

When we use the same parameters as before and hold up the assumption that contracts with penalty agreements are associated with high quality, then the optimal prices if only one contract type is allowed are equal to the situation where both types are possible.

Appendix C: Numeric example

Take for example $\beta = 300, \bar{\lambda} = 600, I = 250, F = 500, c_L = 50, c_H = 150, \pi_L = 0.3, \pi_H = 0.15$. Then the seller has to bear three times the costs in order to halve the failure probability. The maximum potential loss exceeds the benefit gained from the good. So there are buyers who suffer a net loss if the good is defective. Courts don't compensate for the full lost benefit but a penalty can be privately stipulated to cover nearly all losses. Such parameters seem to reflect real-life situations quite well as higher product quality in many cases comes at disproportionately higher costs and courts tend not to compensate for full damages.

Calculating the optimal prices and demand level leads to $p_0^* = 250, \lambda_L^* = \frac{5}{9}, p_F^* = 300, \lambda_H^* = \frac{5}{18}$. In this case nearly customers are served with a total demand of $\frac{15}{18}$ and the price difference is quite distinct. Interestingly there are even some buyers who would suffer a net loss in case of failure who nevertheless opt for the low-price contract. (Because of the assumed uniform distribution the marginal buyer between both types would suffer a loss in the amount of 333.3 and gets only 250 as indemnity in case of a product failure.)

If we only admitted one type of contract then we get the same optimal prices (see Appendix B.2 above) but different total demand. For both contracts being admitted 83.33% of buyers were served. If only the low-price contract was possible, then just 69.44% of the consumers buy the good. And in case of restricting the choice to contracts with penalty agreements the demand amounts to 83.33% again. So again the same number of customers would buy as in the two-contracts case but all of them have to pay the higher price.

Appendix D: Comparative statics

Appendix D.1: Buyers' distribution among contract types

The comparative statics for p_0^* and p_F^* with respect to the parameters c_H, c_L, β, π_L, π_H, I, F are obvious. Therefore we concentrate on λ_L^* and λ_H^* with respect to $\bar{\lambda}$, π_L, π_H as the partial derivatives regarding β, c_H, c_L are again trivial.

$$\frac{\partial \lambda_L^*}{\bar{\lambda}} = -\frac{c_H - c_L}{2(\pi_L - \pi_H)\bar{\lambda}^2} < 0 \tag{40}$$

$$\frac{\partial \lambda_L^*}{\pi_L} = -\frac{c_H - c_L}{2(\pi_L - \pi_H)\bar{\lambda}^2} < 0 \tag{41}$$

$$\frac{\partial \lambda_L^*}{\pi_H} = \frac{c_H - c_L}{2(\pi_L - \pi_H)\bar{\lambda}^2} > 0 \tag{42}$$

$$\frac{\partial \lambda_H^*}{\bar{\lambda}} = \frac{\pi_H(\beta - c_L) - \pi_L(\beta - c_H)}{2\pi_H(\pi_L - \pi_H)\bar{\lambda}^2} < 0 \tag{43}$$

$$\text{because of } \frac{\beta - c_H}{\pi_H} > \frac{\beta - c_L}{\pi_L}$$

$$\frac{\partial \lambda_H^*}{\pi_L} = \frac{c_H - c_L}{2\bar{\lambda}(\pi_L - \pi_L)^2} > 0 \tag{44}$$

$$\frac{\partial \lambda_H^*}{\pi_H} = -\frac{\beta}{2\bar{\lambda}\pi_H^2} + \frac{\pi_L c_H}{2\pi_H^2(\pi_L - \pi_H)\bar{\lambda}} \tag{45}$$

$$\frac{\pi_H^2 c_L + \pi_L c_H(\pi_L - 2\pi_H) - \beta(\pi_L - \pi_H)^2}{2\pi_H^2(\pi_L - \pi_H)^2\bar{\lambda}} =$$

$$\frac{\pi_H^2(c_L - \beta) + \pi_L^2(c_H - \beta) - 2\pi_H\pi_L(c_H - \beta)}{2\pi_H^2(\pi_L - \pi_H)^2\bar{\lambda}} < 0 \forall \beta > c_H$$

$$\text{because of } \pi_L^2 > \pi_H\pi_L$$

The proportion of both buyer groups decreases with larger maximum potential loss. The number of buyers opting for low-price contracts falls with higher probability of failure for low-quality goods but the percentage rises with increasing failure probabilities for high-quality goods since then ceteris paribus buying the high-price contract becomes less attractive. As expected, the opposite holds for the high-price group. It is interesting to note that when we alleviated the assumption of socially optimal high-quality production (i.e. $\beta > c_H$) then the last derivative's sign would be positive for some special cases (e.g. $\beta < c_H \leq \beta + 2\pi_H F$) in which high-quality production would be not socially optimal but still profitable for the seller then higher failure probabilities would lead to higher (!) demand for contracts with penalty clauses. And this is comprehensible since in that special case more buyers want an insurance in form of stipulated fines even if the quality for that high-price contract deteriorates. But as this is only a borderline case we don't consider it any further.

Appendix D.2: Seller's profit

We derive the monopolist's profit using optimal prices and buyer distribution and look how that profit function behaves if either indemnity rewards or penalty amounts are changed.

$$\begin{aligned} V^* &= \lambda_L^*(p_F^*(F), p_0^*(I), F, I)[p_0^*(I) - c_L - \pi_L I] + \\ &\quad \lambda_H^*(p_F^*(F), p_0^*(I), F, I)[p_F^*(F) - c_H - \pi_H F] \end{aligned} \tag{46}$$

$$\begin{aligned} \frac{\partial V^*}{\partial F} &= \underbrace{(p_0^* - c_L - \pi_L I)}_{>0} \underbrace{\left[\underbrace{\frac{\partial \lambda_L^*}{\partial F}}_{-\pi_H} + \underbrace{\frac{\partial \lambda_L^*}{\partial p_F^*} \frac{\partial p_F^*}{\partial F}}_{\pi_H} \right]}_{=0} + \\ &\quad \underbrace{(p_F^* - c_H - \pi_H F)}_{>0} \underbrace{\left[\underbrace{\frac{\partial \lambda_H^*}{\partial F}}_{\pi_L \pi_H} + \underbrace{\frac{\partial \lambda_H^*}{\partial p_F^*}}_{-\pi_L} \underbrace{\frac{\partial p_F^*}{\partial F}}_{\pi_H} \right]}_{=0} + \underbrace{\lambda_H^*}_{>0} \underbrace{\left(\underbrace{\frac{\partial p_F^*}{\partial F}}_{\pi_H} - \pi_H \right)}_{=0} = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} \frac{\partial V^*}{\partial I} &= \underbrace{(p_0^* - c_L - \pi_L I)}_{>0} \underbrace{\left[\underbrace{\frac{\partial \lambda_L^*}{\partial I}}_{\pi_L} + \underbrace{\frac{\partial \lambda_L^*}{\partial p_0^*} \frac{\partial p_0^*}{\partial 0}}_{-\pi_L} \right]}_{=0} + \\ &\quad \underbrace{(p_F^* - c_H - \pi_H F)}_{>0} \underbrace{\left[\underbrace{\frac{\partial \lambda_H^*}{\partial I}}_{-\pi_L \pi_H} + \underbrace{\frac{\partial \lambda_H^*}{\partial p_0^*}}_{\pi_L} \underbrace{\frac{\partial p_0^*}{\partial I}}_{\pi_H} \right]}_{=0} + \underbrace{\lambda_L^*}_{>0} \underbrace{\left(\underbrace{\frac{\partial p_0^*}{\partial I}}_{\pi_L} - \pi_L \right)}_{=0} = 0 \end{aligned} \tag{48}$$

This more complicated way to find the partial derivatives allows us to show how the effects of parameter changes cancel each other out. This explains why the seller's profit remains equal even if the parameters F and I are changed because the seller sets the price such that higher parameter values lead to higher prices. Then in turn the demand adjusts accordingly. So the seller earns the same profit because losing some high-price consumers due to the higher price is compensated by other former low-price customers switching to high-price contracts and thus generating more revenue.

Appendix D.3: Consumers' surplus

The same consideration as for the seller's profit can now be done for the consumer surplus. Therefore we consider that we have two groups of consumers with one demanding low-price goods and the other asking for high-price contracts including a penalty clause. For each group we can calculate the maximum willingness to pay $\bar{p_0} = \beta + \pi_L(I - \lambda)$ and $\bar{p_F} = \beta + \pi_H(F - \lambda)$ from the respective utility functions. So we can write the consumer surplus function again in general form and provide the following derivatives.

$$\begin{aligned} CS &= \int_0^{\hat{\lambda}(p_F^*(F),p_0^*(I),F,I)} \left((\bar{p_0} - p_0^*)\frac{1}{\bar{\lambda}} \right) dt + \\ &\int_{\hat{\lambda}(p_F^*(F),p_0^*(I),F,I)}^{\lambda_F(p_F^*(F),F)} \left((\bar{p_F} - p_F^*)\frac{1}{\bar{\lambda}} \right) dt = \\ &\int_0^{\hat{\lambda}(p_F^*(F),p_0^*(I),F,I)} \left(\frac{\beta - c_L - 2\pi_L t}{2\bar{\lambda}} \right) dt + \\ &\int_{\hat{\lambda}(p_F^*(F),p_0^*(I),F,I)}^{\lambda_F(p_F^*(F),F)} \left(\frac{\beta - c_H - 2\pi_H t}{2\bar{\lambda}} \right) dt = \\ &\frac{1}{2\bar{\lambda}} \left(\hat{\lambda}(c_H - c_L) - \hat{\lambda}^2(\pi_L - \pi_H) + \lambda_F(\beta - c_H - \pi_H \lambda_F) \right) \end{aligned} \tag{49}$$

$$\frac{\partial CS}{\partial F} = \frac{1}{2\bar{\lambda}}[(c_H - c_L) \left(\underbrace{\frac{\partial \hat{\lambda}}{\partial F}}_{-\pi_H} + \underbrace{\frac{\partial \hat{\lambda}}{\partial p_F^*} \frac{\partial p_F^*}{\partial F}}_{\pi_H} \right) - \tag{50}$$

$$(\pi_L - \pi_H)\left(2\hat{\lambda}\left(\underbrace{\frac{\partial\hat{\lambda}}{\partial F} + \frac{\partial\hat{\lambda}}{\partial p_F^*}\frac{\partial p_F^*}{\partial F}}_{=0}\right)\right) +$$

$$(\beta - c_H - \pi_H\lambda_F)\left(\underbrace{\frac{\partial\lambda_F}{\partial F}}_{\pi_H} + \underbrace{\frac{\partial\lambda_F}{\partial p_F^*}\frac{\partial p^* F}{\partial F}}_{-\pi_H}\right) +$$

$$\lambda_F\left(-\pi_H\left(\underbrace{\frac{\partial\lambda_F}{\partial F} + \frac{\partial\lambda_F}{\partial p_F^*}\frac{\partial p^* F}{\partial F}}_{=0}\right)\right)] = 0$$

$$\frac{\partial CS}{\partial I} = \frac{1}{2\bar{\lambda}}[(c_H - c_L)\left(\underbrace{\frac{\partial\hat{\lambda}}{\partial I}}_{\pi_L} + \underbrace{\frac{\partial\hat{\lambda}}{\partial p_0^*}\frac{\partial p_0^*}{\partial I}}_{-\pi_L}\right) - \quad (51)$$

$$(\pi_L - \pi_H)\left(2\hat{\lambda}\left(\underbrace{\frac{\partial\hat{\lambda}}{\partial I} + \frac{\partial\hat{\lambda}}{\partial p_0^*}\frac{\partial p_0^*}{\partial I}}_{=0}\right)\right)] = 0$$

Again we have the same situation that the direct effect of parameter changes and the indirect effect (through changes in the optimal prices) cancel each other out. So varying the parameters F or I doesn't affect consumer surplus.

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